Reduced Order Modelling (250971)

General Information

School	ETSECCPB
Departments	Departament d'Enginyeria Civil i Ambiental (DECA)
Credits	5.0 ECTS
Programs	MÀSTER UNIVERSITARI EN MÈTODES NUMÈRICS EN ENGINYERIA (pla 2012) MÀSTER UNIVERSITARI EN MÈTODES NUMÈRICS EN ENGINYERIA (pla 2012)
Course	2025/26

Main teaching language at each group

- Group 10Q1 English (Q1)

Faculty

Responsible Faculty: Pavel Ryzhakov Barbanel
Faculty: Riccardo Rossi Bernecoli, Pavel Ryzhakov Barbanel, Alireza Shamekhi, Nicolás Sibuet Ruiz, Sergio Zlotnik Martinez

Objectives of Education

The students will acquire the knowledge of the techniques of reduction of models of types a posteriori and a priori. They will apply the different techniques of ROM (Reduced order modeling) to general data problems, as well as to computational mechanics problems.

1. Ability to develop and use reduced order models for engineering applications.
2. Ability to computationally implement the different ROM techniques.
3. Ability to select the most appropriate technique for each problem.

Singular Value Decomposition (SVD), Modal Analysis, Proper Orthogonal Decomposition, Hyperreduction, Proper Generalized Decomposition, Reduced bases, Response surfaces.

Total hours of student work

		Hours	Percentage
Supervised Learning	Large group	45.0 h	100.00 %
	Medium group	0.0 h	0.00 %
	Laboratory classes	0.0 h	0.00 %
	Guided Activities	0.0 h	0.00 %
Self Study		80.0 h	

Contents

Math preliminaries

Vector and matrix norms (useful when defining the optimization objectives), rank, column space, span of the column.

Eigenvalues and Eigenvectors. Types of matrices and their properties (symmetry, orthogonality, orthonormality).
Mathematical Preliminaries: practical session

Specific Objectives

This section will serve as a brush up on the necessary Linear Algebra concepts required for the understanding of model reduction when motivated as optimization problems.

Data and SVD

This section will introduce different types of data and its representation in matrix form. The data will be treated in a broad sense: experiments, images, numerical simulations etc. The singular value decomposition (SVD) will be introduced as a means for finding the most relevant intrinsic patterns in these data.
Data representation, PCA, Gappy Data

Specific Objectives

Familiarize the students with the application of SVD using programming environment.

POD ("proper orthogonal decomposition")

Introduction to POD: SVD applied to PDEs. Dimensionality reduction as a means of representing spatio-temporal behavior of complex systems using low-dimensional approximations ("low-dimensional patterns of dynamic activity").

POD basics: PDEs, separation of variables. Representation of the spatial distribution of the variable using modal basis. Galerkin projection using orthonormal basis functions. Ways of selecting the basis.

Application of POD to FEM model.

Modal analysis

Modal Analysis: an "a priori" alternative to POD. Differences and similarities. Application to modeling of a linear structure.
Modal analysis: practical session
An alternative approach to ROM is the a priori approach, with no need of a prior sampling the parametric space and a posterior solving of the reduced equation. The idea of the PGD is based on considering parameters as extra coordinates and handling the high-dimensionality by using separated approximations.

Hyperreduction

Hyperreduction: a means for dimensionality reduction of non-linear problems.

Assignments presentation

Parametric problems

The lecture aims at presenting the parametric problem (full order) to be used as a demonstrator for all the concepts introduced in the second part of the course. The set of parametric solutions is analyzed as a low-order manifold in a high-dimension Euclidean space. Different techniques to describe the set of solutions are quickly reviewed.

Reduced basis

The full-order model is transformed into a reduced-order model using a low-dimensional representation computed offline. Attention is paid to different constructions of the reduced bases, and their comparison. Either following a greedy technique (adding a new term only if it is nor redundant) or orthogonalizing the basis.

Reduced bases and a posteriori ROM: practical session

PGD

The PGD philosophy goes far beyond the solution of the parametric problem and allows building explicit surrogates for QoI described as simple operations of the solution. The lecture aims at producing the surrogates corresponding to different QoI using the available PGD operations.

Response surfaces

The simplest approach to build a surrogate model for any parametric QoI is reconstructing the response function as a multiparametric field. Classical interpolation and functional fitting techniques are reviewed, paying special attention to the offline sampling. The sensitivity of the obtained solutions to the choice of location of the sampling points is also analyzed.
Response surfaces: practice

Teaching Methodology

The course consists of 1.2 hours per week of classroom activity (large size group) and 1.2 hours weekly with half the students (medium size group).

The 1.2 hours in the large size groups are devoted to theoretical lectures, in which the teacher presents the basic concepts and topics of the subject, shows examples and solves exercises.

The 1.2 hours in the medium size groups is devoted to solving practical problems with greater interaction with the students. The objective of these practical exercises is to consolidate the general and specific learning objectives.

The rest of weekly hours devoted to laboratory practice.

Support material in the form of a detailed teaching plan is provided using the virtual campus ATENEA: content, program of learning and assessment activities conducted and literature.

Although most of the sessions will be given in the language indicated, sessions supported by other occasional guest experts may be held in other languages.

Grading Rules

(*) The evaluation calendar and grading rules will be approved before the start of the course.

Assistance is mandatory. The grade for the course is obtained in the continuous assessment mode.

Continuous assessment consists of doing different activities ("assignments") of didactic nature, carried out during the course (in the classroom and at home) as well as an examination.

The examination will consist of a part with questions about theoretical concepts associated with the learning objectives of the subject, and/or a practical part related to the exercises and assignments.

Office Hours

It will be specified on the course page.

Bibliography

Basic

- Brunton, S.L.; Kutz, J.N. Data-driven science and engineering: machine learning, dynamical systems, and control. Second edition. Cambridge: Cambridge University Press, 2022. ISBN 9781009098489.