

Finite Elements in Fluids (250957)

General Information

School	ETSECCPB
Departments	Centre Específic de Recerca de Mètodes Numèrics en Ciències Aplicades i Enginyeria (CER-LACÀN-UPC) Departament d'Enginyeria Civil i Ambiental (DECA)
Credits	5.0 ECTS
Programs	MÀSTER UNIVERSITARI EN MÈTODES NUMÈRICS EN ENGINYERIA (pla 2012) MÀSTER UNIVERSITARI EN MÈTODES NUMÈRICS EN ENGINYERIA (pla 2012)
Course	2025/26

Main teaching language at each group

- Group 10Q2 English (Q2)

Faculty

Responsible Faculty: Matteo Giacomini

Faculty: Valeria Agustina Felipe Ramudo, Matteo Giacomini, Antonio Huerta Cerezuela

Objectives of Education

The module covers the essential theoretical and practical aspects of the numerical approximation of partial differential equations modeling flow problems, specifically focusing on Euler and Navier-Stokes equations.

Learning objectives: be able to understand the fundamentals of mathematical modeling the motion of fluids, construct numerical algorithms to simulate convection-diffusion-reaction phenomena, understand the numerical difficulties and basic methods to approximate incompressible flows (Stokes and Navier-Stokes equations) and purely convective inviscid flows (Euler equations). More specifically:

- describe, predict and formulate techniques for steady linear scalar convection-diffusion-reaction problems,
- understand stabilization techniques for convection-dominated flows,
- formulate techniques for inviscid flows,
- understand the major difficulties in handling hyperbolic equations and the concept of well-posed Euler problem,
- describe, predict and formulate techniques for incompressible flows,
- understand the major difficulties in solving Stokes and Navier-Stokes equations.

Competencies

Especific

Practical numerical modeling skills. Ability to acquire knowledge on advanced numerical modeling applied to different areas of engineering such as: civil or environmental engineering or mechanical and aerospace engineering or bioengineering or Nanoengineering and naval and marine engineering, etc..

Knowledge of the state of the art in numerical algorithms. Ability to catch up on the latest technologies for solving numerical problems in engineering and applied sciences.

Materials modeling skills. Ability to acquire knowledge on modern physical models of the science of materials (advanced constitutive models) in solid and fluid mechanics.

Experience in numerical simulations. Acquisition of fluency in modern numerical simulation tools and their application to multidisciplinary problems engineering and applied sciences.

Interpretation of numerical models. Understanding the applicability and limitations of the various computational techniques.

Experience in programming calculation methods. Ability to acquire training in the development and use of existing computational programs as well as pre and post-processors, knowledge of programming languages and of standard calculation libraries.

Total hours of student work

		Hours	Percentage
Supervised Learning	Large group	45.0 h	100.00 %
	Medium group	0.0 h	0.00 %
	Laboratory classes	0.0 h	0.00 %
	Guided Activities	0.0 h	0.00 %
Self Study		80.0 h	

Contents

Review of basic concepts

Description of the flow motion equations. Weak form. Discretization. Elementary matrices and assembly. Numerical integration. Reference element. Implementation.

Steady convection-diffusion problems

Analysis of the convection-diffusion equation 1D. Effect of the Péclet number and need for stabilization. Stabilization techniques. Consistent stabilized formulations. Extension to 2D and 3D. Practical exercises and implementations.

Pure convection problems

Notion of pure convection. Characteristics lines. Classical techniques of time and space discretization: theta-methods and Galerkin discretization. Taylor-Galerkin discretizations. Stability and accuracy. Effect of the Courant number. Practical exercises and implementation.

Compressible flow problems

Nonlinear hyperbolic PDEs. Burgers' equation. Shock, rarefaction, and contact waves. Entropy solutions. Basic properties of Euler equations. Boundary conditions for hyperbolic PDEs. Riemann solvers. Introduction to discontinuous Galerkin methods. Practical exercises.

Viscous incompressible flow problems

Stokes equation. Saddle point problem. LBB condition. Stable and unstable spatial discretizations. Stabilizations. Consistent stabilized formulations. Steady Navier-Stokes. Linearization techniques. Effect of the Reynolds number. Stabilizations. Transient Navier-Stokes. Time discretization. Algebraic splitting and fractional step. Finite volume methods. Introduction to turbulence modelling. Practical exercises and implementation.

Discontinuous Galerkin methods

Basic concepts on modern discontinuous Galerkin methods. Interior Penalty DG. Compact DG. Local DG. Hybridizable DG. Formulation and advantages of HDG for flow problems. Practical exercises and implementation.

Teaching Methodology

The module consists of 0,6 hours per week of classroom activity (large size group) and 1,2 hours weekly with half the students (medium size group).

The 0,6 hours in the large size groups are devoted to theoretical lectures, in which the teacher presents the basic concepts and topics of the subject, shows examples and solves exercises.

The 1,2 hours in the medium size groups is devoted to solving practical problems with greater interaction with the students. The objective of these practical exercises is to consolidate the general and specific learning objectives.

The rest of weekly hours devoted to laboratory practice.

Support material in the form of a detailed teaching plan is provided using the virtual campus ATENeA: content, program of learning and assessment activities conducted and literature.

Grading Rules

() The evaluation calendar and grading rules will be approved before the start of the course.*

The grade of the course is obtained from a continuous assessment during the module. This consists of several activities, both individual and in group, of incremental training, carried out during the module, both in and out of the classroom.

The final grade will be computed as follows:

- 40% written exam on the first part of the module (Test 1);
- 40% written exam on the second part of the module (Test 2);
- 20% classwork (Periodic assignments on practical and programming exercises).

Students who participated in exam 1 can opt to disregard its evaluation by replacing the second test with an exam covering the entire syllabus of the module. In this case, the final grade will be computed as follows:

- 80% written exam on the entire module;
- 20% classwork (Periodic assignments on practical and programming exercises).

The two grading modalities described above are mutually exclusive. Students who opt for the exam covering the entire syllabus must explicitly request it to the referent professor at least two weeks prior to the date of exam 2.

Attendance of two written exams is mandatory.

The written tests will assess the assimilation of the fundamental concepts related to the learning objectives of the module and will consist of:

- theoretical questions on the numerical methods presented in class;
- practical exercises requiring to write the discrete formulation for a given method and problem;
- interpretation questions commenting on the expected performance of the methods starting from the theory.

The evaluation of the classwork will assess the incremental learning of the students and will be based upon:

- periodic assignments consisting of both written and programming exercises on the numerical methods seen during the module, to be submitted for correction;
- participation during lectures, exercise and practical classes.

For the distance learning version of the Master, classwork evaluation will only consider the submitted assignments.

Test Rules

The assignments must be submitted via ATENeA by the announced deadline. Late submissions or assignments submitted using other means will not be accepted and will be graded 0.

The assignments must be performed individually: students are encouraged to discuss about the assignments but the submitted work must be the result of one own efforts. Plagiarism in the assignments will be punished with a 0 in the classwork grade.

The written exams must be performed individually. Exams are closed-book. Books and lecture notes are not

allowed. Students are allowed to bring one A4 sheet, handwritten on one of the faces of the paper, with formulae. Plagiarism will be punished with a 0 in the module grade.

Office Hours

Upon appointment by email.

Bibliography

Basic

- Donea, J.; Huerta, A. [Finite element methods for flow problems](#). Chichester: John Wiley & Sons, 2003. ISBN 0471496669.

Resources

Teaching material including slides, class notes, lists of exercises are available on ATENeA.

The reference textbook available in UPC library is:

Donea, J., Huerta, A., Finite Element Methods for Flow Problems, Wiley, 2003