

# Continuum Mechanics (250952)

## General Information

<b>School</b>	ETSECCPB
<b>Departments</b>	Departament d'Enginyeria Civil i Ambiental (DECA)
<b>Credits</b>	5.0 ECTS
<b>Programs</b>	MÀSTER UNIVERSITARI EN ENGINYERIA ESTRUCTURAL I DE LA CONSTRUCCIÓ (pla 2015) MÀSTER UNIVERSITARI EN MÈTODES NUMÈRICS EN ENGINYERIA (pla 2012) MÀSTER UNIVERSITARI EN MÈTODES NUMÈRICS EN ENGINYERIA (pla 2012)
<b>Course</b>	2024/25

## Main teaching language at each group

- Group 10EN1 English (Q1)

## Faculty

Responsible Faculty: Javier Bonet Carbonell

Faculty: Carlos Agelet De Saracibar Bosch, Javier Bonet Carbonell, Ramon Codina Rovira, Oriol Lloberas Valls

## Objectives of Education

This is a complete course in nonlinear continuum mechanics for engineers. It carries out a deep review of the fundamental concepts, including motion, deformations, strains, stresses, governing laws of balance, variational principles and an introduction to the Mechanics of solids and of fluids.

\* The students will be able to understand and assimilate the foundations of the mechanics of solids, identifying the most important aspects of material modeling, and dissipation mechanisms associated with nonlinear behaviour. \* They have to be able to interpret the physical meaning of the material properties and properly identify the numerical methods for the solution of problems of solid mechanics, with its application to elasticity, and learn the foundations of fluid mechanics. \* The students will develop practical skills to work with tensors and formulate and develop analysis of diverse problems of solids in engineering.

\* Tensor algebra (definitions, invariants, gradients, divergences, rotational, integral theorems).

\* Kinetics: deformation<sup>00</sup> and strain (strain tensors).

\* Small deformations and compatibility.

\* Stress tensors.

\* Governing laws

\* Constitutive Laws (laws of thermodynamics, deformation energy, elasticity)

\* Boundary value problems in linear elasticity (2D)

\* Introduction to plasticity (Von Mises, Tresca, Mohr, Coulomb)

\* Ideal and potential flows.

\* Viscous incompressible flow (with an introduction to turbulent flow)

\* Learning resources:

o Bonet, J, Gil AJ, Wood RD, Nonlinear Solid Mechanics for Finite Element Analysis - Statics, Cambridge University Press

o Bonet, J, Gil AJ, Wood RD, Nonlinear Solid Mechanics for Finite Element Analysis - Dynamics, Cambridge University Press

o Holzapfel, G.A., Nonlinear solid mechanics, a continuum approach for engineering, Wiley, 2000  
 o Currie,

The main objectives of the course are the presentation, understanding and mastery of the basic fundamentals of nonlinear continuum mechanics and their application to solid mechanics and fluid mechanics.

## Total hours of student work

		Hours	Percentage
Supervised Learning	Large group	25.5 h	56.67 %
	Medium group	9.75 h	21.67 %
	Laboratory classes	9.75 h	21.67 %
	Guided Activities	0.0 h	0.00 %
Self Study		80.0 h	

## Contents

### Tensor Algebra

In this chapter, the basic notation used in the course and a comprehensive review of the main concepts of tensor algebra is introduced.

Contents:

- Introduction
  - Vector Algebra
  - Tensor Algebra
  - Higher-order tensors
  - Differential Operators
  - Integral Theorems
- Problems

### Specific Objectives

The objectives are to introduce the tensor notation to be used in the course and do a review of tensor algebra.

### Motion

This topic describes the main assumptions of the Mechanics of Continuous Media and the main concepts of non-linear kinematics of particles are introduced: equation of motion, description of materials and spatial properties of the continuum derived materials, space and convective, displacement, velocity, acceleration, trajectories, streamlines, materials and spatial surfaces and volumes material espacial.

Contents:

- Definition of Continuous Medium
  - Motion equation
  - Material and spatial descriptions
  - Material and spatial time derivatives
  - Displacement
  - Velocity
  - Accelerations
  - Trajectories
  - Streamlines
  - Material and spatial surface
  - Material and spatial volume
- Problems

## Specific Objectives

The aim is to introduce the main concepts of nonlinear kinematics of particles.

## Strains

In this chapter the main aspects of nonlinear kinematics of deformation of a continuum medium are introduced.

Contents:

- Deformation gradient tensor
- Material and spatial gradient of displacements tensors
- Green-Lagrange and Almansi strain tensors
- Volumetric Strain
- Variation of the area
- Polar decomposition of the deformation gradient tensor
- Stretching
- Variation of angles
- Spatial gradient of velocity tensor
- Deformation rate and rotation rate tensors
- Material time derivative of different tensors

Problems

## Specific Objectives

The aim is to introduce the main concepts and tension associated with the deformation of a continuous medium.

## Infinitesimal strains

This topic hypothesis under infinitesimal strains are introduced and the corresponding simplified expressions used in the context of infinitesimal deformations are obtained. The concept of compatibility equations for the infinitesimal strain tensor is also introduced.

Contents:

- Assumptions of the theory of infinitesimal deformations
- Tensor displacement gradient
- Infinitesimal strain tensor
- Variation of infinitesimal volume
- Polar decomposition
- Stretching
- Variation of angles
- Matrix notation
- Compatibility Equations

Problems

## Specific Objectives

The main objectives are to introduce the hypothesis of infinitesimal deformation theory and particularizing tensors introduced in the nonlinear case, the case of infinitesimal deformations.

## Stresses

This topic describes the concepts of forces and stresses on a continuous medium.

Contents:

- Body forces and surface forces
- Cauchy's theorems
- Cauchy stress tensors and first Piola-Kirchhoff stress tensor
- Piola transformation and Piola identity
- Kirchhoff and second Piola-Kirchhoff stress tensors

## Specific Objectives

The main objectives are to introduce the concepts of forces and stress tensors.

## Balance/conservation equations

This topic describes the basic principles of conservation / balance of a continuous medium are introduced, global / local shape and material / spatial. The ultimate goal is to obtain the governing equations of a problem of continuum mechanics.

Contents:

- Basic equations of conservation / balance
  - Conservation of Mass
  - Convective flux of a property
  - Lemma of Reynolds
  - Reynolds Transport Theorem
  - Balance of momentum
  - Balance of moment of momentum
  - Thermodynamics
  - First law of thermodynamics. Energy Balance
  - Second Law of Thermodynamics
  - Thermodynamic Processes
  - Governing Equations
  - Uncoupled mechanical and thermal problems
- Problems

## Specific Objectives

The main objectives are to introduce the basic principles of conservation / balance and get in space and equipment locally, the governing equations of a problem in continuum mechanics.

## Linear elasticity

This topic describes the main concepts of linear elasticity are presented, leading to pose and solve the linear elastic problems.

Contents:

- Linear elastic model
  - Linear Elastic Problem
  - Solution of linear elastic problem
  - Orthogonal curvilinear coordinates
- Problems

## Specific Objectives

The main objectives are to introduce the constitutive equations for linear elastic model and come to formulate and solve the linear elastic problem.

## Fluid Mechanics

This chapter introduces additional concepts needed to formulate a generic problem in fluid mechanics.

Contents:

- Introduction
- Constitutive Equations
- Governing Equations

## Specific Objectives

The main objective is to introduce the constitutive equations for a fluid mechanics problem.

## Newtonian fluids

In this chapter, the constitutive equation for a Newtonian fluid is introduced and the formulation and solution of a fluid dynamics problem for Newtonian fluids is shown.

content:

- Constitutive equation
  - Governing Equations
  - Boundary Conditions
  - Orthogonal curvilinear coordinates
- Problem

### **Specific Objectives**

The main objectives are to introduce the constitutive equations for a Newtonian fluid and get to pose and solve a problem in fluid mechanics for a Newtonian fluid.

## **Activities**

### **Tensor Algebra Exercises**

**Dedication**

1h

### **Exercises topic: Particle Motion**

**Dedication**

1h

### **Exercises topic: Strains**

**Dedication**

1h

### **Exercises theme infinitesimal strains**

**Dedication**

1h

### **Exercises topic: Basic equations of conservation / balance**

**Dedication**

1h

### **Exercises Theme: Linear Elasticity**

**Dedication**

1h

### **Exercises topic: Newtonian Fluids**

**Dedication**

1h

## **Teaching Methodology**

The course consists of 40 hours of class, taught semi-intensively, intensively from Monday to Friday over the first two weeks and then one day per week.

The classes include theory, problems and guided activities. For each subject, the necessary theoretical concepts are first introduced, then some examples and exercises are solved by the professor. The students will solve some additional problems during the class time under the supervision of the professor. On the other hand, students will solve some additional assignments as homework.

The student has a basic teaching material for monitoring and understanding of the course: (1) the files in pdf format with the updated content of the classes; (2) video recordings of classroom lectures given during 2012-2013; basic and supplementary bibliographic references.

Although most of the sessions will be given in the language indicated, sessions supported by other occasional guest experts may be held in other languages.

## Grading Rules

*(\*) The evaluation calendar and grading rules will be approved before the start of the course.*

The course grade is obtained from the ratings of the continuous assessment (30%) and final exam (70%).

Continuous assessment: The student has to solve along the course and supervised by the professor, several exercises and problems, both in the classroom (during school hours), and beyond.

Final exam: The final exam consists of some questions and problems similar to those that have been raised and resolved in class.

## Test Rules

Continuous assessment: Failure to perform a continuous assessment activity in the scheduled dates will result in a zero mark in that activity.

Final exam: The final exam will be an open book exam.

## Office Hours

Negotiable

## Bibliography

### Basic

- Bonet, J, Gil, AJ, Wood RD. Nonlinear Solid Mechanics for Finite element Analysis - Statics. Cambridge University Press,
- Bonet, J.; Wood, R.D. [Nonlinear continuum mechanics for finite element analysis](#). Cambridge: Cambridge University Press, 2008. ISBN 9780521838702.
- Bonet, J, Gil AJ, Wood RD. Nonlinear Solid Mechanics for Finite Element Analysis - Dynamics.

### Complementary

- Holzapfel, G.A. [Nonlinear solid mechanics: a continuum approach for engineering](#). Chichester: John Wiley & Sons, 2000. ISBN 0471823198.
- Oliver, X.; Agelet de Saracibar, C. [Continuum mechanics for engineers: theory and problems](#). 2nd ed. Barcelona: els autors, 2017.
- Oliver Olivella, X.; Agelet de Saracibar, C. [Mecánica de medios continuos para ingenieros](#). 2a ed. Barcelona: Edicions UPC, 2002. ISBN 848301582X.
- González, O.; Stuart, A.M. [A first course in continuum mechanics](#). Cambridge: Cambridge University, 2008. ISBN 9780521886802.
- Marsden, J.E.; Huges, T.J.R. [Mathematical foundations of elasticity](#). New York: Dover, 1994. ISBN 0486678652.

- Truesdell, C.; Noll, W. [The non-linear field theories of mechanics](#). 3rd ed. Berlin: Springer-Verlag, 2004. ISBN 3540027793.



Escola de Camins