Differential Geometry and Differential Equations (250122)

General Information

School	ETSECCPB
Departments	Departament d'Enginyeria Civil i Ambiental (DECA) Escola Tècnica Superior d'Enginyeria de Camins, Canals i Ports de Barcelona (ETSECCPB)
Credits	9.0 ECTS
Programs	GRAU EN ENGINYERIA CIVIL (pla 2017) GRAU EN ENGINYERIA CIVIL (pla 2010)
Course	2022/23

Main teaching language at each group

- Group 10Q1 Spanish (Q1)
- Group ENGQ1 English (Q1)

Faculty

Responsible Faculty: Enrique Bendito Perez, Matteo Giacomini
Faculty: Marino Arroyo Balaguer, Enrique Bendito Perez, Matteo Giacomini

Objectives of Education

Students will acquire an understanding of differential geometry (including curves and surfaces, as well as integration on manifolds and integral theorems) and partial differential equations of mathematical physics. They will also develop the skills to analyse and solve mathematical problems in engineering that involve these concepts.

Upon completion of the course, students will have acquired the ability to: 1. Relate partial differential equations to engineering problems in continuous media. 2. Program complex solutions using basic software and obtain numerical solutions. 3. Develop analytical solutions to complex multidimensional boundary value and initial value problems with simple geometric conditions that allow an analysis of these solutions, including a parametric study. 4. Carry out an analytical description of curves and surfaces, calculate their properties, and perform differential and integral calculus operations on them.

Basic tools in metric geometry: Ruler-and-compass constructions and demonstrations; Floor plans; Technical drawing; The conic system

Competencies

Especific

Ability to provide analytical descriptions of curves and surfaces, calculate their properties and perform differential calculus operations on them; find analytical solutions to complex contour and initial value problems in various dimensions and with simple geometrical conditions enabling an analysis, including a parametric study, to be made of these solutions

Transversal

SUSTAINABILITY AND SOCIAL COMMITMENT - Level 1. Analyzing the world¿s situation critically and systemically, while taking an interdisciplinary approach to sustainability and adhering to the principles of

sustainable human development. Recognizing the social and environmental implications of a particular professional activity.
EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 1. Planning oral communication, answering questions properly and writing straightforward texts that are spelt correctly and are grammatically coherent.
EFFECTIVE USE OF INFORMATI0N RESOURCES - Level 2. Designing and executing a good strategy for advanced searches using specialized information resources, once the various parts of an academic document have been identified and bibliographical references provided. Choosing suitable information based on its relevance and quality.
SELF-DIRECTED LEARNING - Level 2: Completing set tasks based on the guidelines set by lecturers. Devoting the time needed to complete each task, including personal contributions and expanding on the recommended information sources.

Total hours of student work

		Hours	Percentage
Supervised Learning	Large group	30.0 h	30.30 %
	Medium group	30.0 h	30.30 %
	Laboratory classes	30.0 h	30.30 %
	Guided Activities	9.0 h	9.09 %
Self Study		126.0 h	

Contents

CHANGE OF VARIABLES

To demonstrate the usefulness of the Inverse Function Theorem
Describe the most interesting curvilinear coordinates in the two-and three-dimensional Euclidean space
Consideration of vector spaces, of the same dimension of the object coordinate, in each of its points.
Construction of the bases of vector spaces as tangent vectors to the coordinate curves

Specific Objectives

Understand the concept of change of variables respecting the regularity properties
Learn to make the most usual change of variable
Describe vector fields related to coordinate objects
Understanding the Jacobian of curvilinear coordinates as the change of variable matrix for vector fields

PARAMETRIZED CURVES

Change of parameter and tangent to curves
Qualitative understanding of regular curves

Specific Objectives

Know the different kinematic concepts related to the trajectory of a point in geometric terms
Construct the minimum information necessary to distinguish the curves parameterized except for rigid motion

GEOMETRY OF SURFACES

Understanding the structure of surfaces and the control of their regularity
Determine the surfaces and their aspect by the way of to describe
Diffeomorphisms on surfaces and the First Fundamental Form
The gaussian curvature and other curvatures in coordinates

Specific Objectives

Distinguish why an application is a surface and evaluate when it has a tangent plane
Knowing ruled surfaces, surfaces of revolution and level surfaces
Consider the composition with the parametrizations to extend the differential calculus on the surfaces and introduce the ability to measure on surfaces
Get the curvatures of any parameterized surface and understand the major characteristics of the surfaces most used

MANIFOLDS WITH BOUNDARY

Surface extension of the concept of regular and metric dimension n objects in a space of dimension s environment
Consider parameterizations of closed intervals and closed pseudo-intervals, and characterize the orientation induced on the boundary of the parameterization

Specific Objectives

Provide a common language to curves, surfaces, and open two-and three-dimensional
Know the description of a variety of objects such as finite union of pseudo-intervals or for the parameterization of these

STOKES THEOREM

Obtaining the StokesTheorem on parameterized intervals, by applying the Barrow Rule in each variable
Getting the divergence theorem, Stokes-Ampère and Green in the plane, as elementary application of general Stokes theorem
Solving exercises in which they appear most distinctive resources that facilitate the resolution of a wide range of problems

Specific Objectives

Understanding the scope of integration of functions and the correspondence between the implementation of integration within the objects or on their boundary
Understanding the relationship between divergence, flow and circulation of vector fields on tri and bidimensional objects and their boundaries
Learning to solve problems of integration on complex objects, but whose pieces are described by simple expressions that facilitate the realization of calculations on them

GEOMETRY EXAM

VARIATIONS OF INTEGRALS

Knowledge of the variation of integrals with respect to the time when both subintegral function as the domain of integration depend of the time
Getting some conservation laws, as pure application of the results of integration and basic mechanical concepts in elementary media

Specific Objectives

Using elementary techniques of integration and implementation of the classic theorems, connect with key elements in the description of continuous media
Spending on basic physical laws from a simplified perspective emphasizing techniques of vector calculus

CALCULUS OF VARIATIONS

Formulation of the problem, necessary condition for extrema, Euler and Euler-Lagrange equations and natural boundary conditions
Systematically explore the problems posed by the necessary condition for extrema, identifying the characteristics of the different types of equations obtained, self-adjoint problems, problems of one or more variables, eigenvalues, natural boundary conditions or enforced

Specific Objectives

It is intended to provide a systematic way of expose problems, covering the most relevant part of the problems that arise in the context of differential equations in both one variable as in several variables
It is intended to come into contact with a number of problems that will gradually expand the horizon of action, from the classic problem of brachistochrone to the equations from the minimization of elastic strain energy

MATHEMATICAL PHYSICS EQUATIONS

From the minimization of quadratic functionals and considering isoperimetric conditions, with functions of one variable, we obtain linear boundary problems and eigenvalues problems for ODE's
Will be posed classification criteria and the prototypes of elliptic equations, parabolic equations and hyperbolic equations

Specific Objectives

Aims to provide tools to solve these problems of great interest in applications, the tools previously known consist only in rudiments to address basic problems of initial value
We present families of problems that can be tackle, heat equations, wave equation, Laplace equation, types of domains and the initial and boundary conditions

QUADRATIC FUNCTIONALS

Estimation and analysis of eigenvalues of Sturm-Liouville problems, and solving boundary value problems including the use of Green's functions

Specific Objectives

Gain experience in using methods of solving such problems, these techniques remain in force when dealing with problems of partial differential equations by separation of variables

SEPARATION OF VARIABLES

We introduce the Fourier method for the one-dimensional diffusion problems, and we extend this method of resolution to any problem that fits with it. This allows us to resolve many issues previously raised whose solutions will be represent in orthonormal bases

Specific Objectives

Have a tool to obtain effective analytical solutions of problems of interest in many applied areas and also understand its scope, on the other hand it allows to assess the need for other techniques to address problems that can not solve the method

APPROXIMATED SOLUTIONS

The direct methods consist in considering Minimizing sequences and are not easy to systematize, the weak form is to write the equations in integral form and with less regularity requirements and are easy to systematize
Expose a method to control the coefficients of the existing problems, to obtain the approximate solution in finite dimensional spaces. Evolution problems.

Specific Objectives

Consider alternative methods to the separation of variables for the solution of problems of great interest and expose the rudiments to the weak form as the way for implementation the approximation methods for many problems.
Access the basic rudiments so that, throughout their studies, they can know the methods of numerical solution of many problems of interest including evolution problems.

EQUATIONS EXAM

Activities

Parameterization and calculus on a real object

It proposes the realitation of a individual project, namely the choice of a real object, its reasonable idealization for their parametrization, graphical representation and the resolution of a small number of questions of vector calculus about it. All directed and revised on the lab classes. The project will end with an individual oral presentation of a summary in powerpoint

Dedication

4h 30m

Study and solve a problem related to engineering PDEs

It proposes the creation of a individual project in the field of differential equations, consisting in choosing and solving a problem of mathematical physics related to the Civil Engineering studies. It is intended that students internalize that many physical phenomena can be modeled by differential equations and to take responsibility in the formulation of the problem for motivate it to know the possible methods of resolution. All directed and revised in the lab classes. The project will end with a presentation oral of a powerpoint summary

Dedication

4h 30m

Teaching Methodology

The course consists of 6 hours per week of classroom to classroom (large group).

They are devoted to lectures 2.5 hours largest group, the teacher presents the basic concepts and materials of matter, presents examples and exercising.

It is dedicated 3.5 hours largest group, solving problems with greater interaction with the student. Practical exercises to consolidate the objectives of general and specific learning.

The rest of weekly hours dedicated to evaluations.

Support material is used in the form of detailed teaching plan using the virtual campus ATENEA: content, programming and evaluation activities directed learning and literature.

Although most of the sessions will be given in the language indicated, sessions supported by other invited experts may be held in other languages. The language may change due to force majeure.

Grading Rules

(*) The evaluation calendar and grading rules will be approved before the start of the course.

There will be two written exams of the subject. One in Differential Geometry area, G1, and other in Differential Equations area, E1. The dates of the two exams will be located within the period determined by the School.
The exams will have the same value for the final note

The final note will, NF = (G1 + E1) / 2.

Criteria for re-evaluation qualification and eligibility: Students that failed the ordinary evaluation and have regularly attended all evaluation tests will have the opportunity of carrying out a re-evaluation test during the period specified in the academic calendar. Students who have already passed the test or were qualified as non-attending will not be admitted to the re-evaluation test. The maximum mark for the re-evaluation exam will be five over ten (5.0). The non-attendance of a student to the re-evaluation test, in the date specified will not grant access to further re-evaluation tests. Students unable to attend any of the continuous assessment tests due to certifiable force majeure will be ensured extraordinary evaluation periods.

These tests must be authorized by the corresponding Head of Studies, at the request of the professor

responsible for the course, and will be carried out within the corresponding academic period.

Evaluation in the English Group

The evaluation will consist of three elements. 30% of the grade will depend on activities performed during classes. These will include short evaluations of assigned reading, exercises performed individually or in group, and active participation. There will be two exams in the periods set by the Civil Engineering school for the mid-term and the final evaluations, each accounting for 35% or the grade.

Students that have participated in the activities associated to the ordinary evaluation but not passing the course will be offered a re-evaluation. The maximum mark for the re-evaluation exam will be five over ten (5.0). The non-attendance of a student to the re-evaluation test, in the date specified will not grant access to further re-evaluation tests. Students unable to attend any of the continuous assessment tests due to certifiable force majeure will be ensured extraordinary evaluation periods.

Test Rules

Failure to perform a continuous assessment activity in the scheduled period will result in a mark of zero in that activity.

Office Hours

To be arranged in the course.

Bibliography

Basic

- Carmo, M.P. Geometría diferencial de curvas y superficies. Madrid: Alianza, 1990. ISBN 8420681350.
- Marsden, J.E.; Tromba, A.J. Cálculo vectorial. 5a ed. Madrid: Addison Wesley, 2004. ISBN 8478290699.
- Raviart, P.-A.; Thomas, J.-M. Introduction à l'analyse numérique des équations aux dérivées partielles. Paris: Dunod, 1998. ISBN 9782100486458.
- Haberman, R. Ecuaciones en derivadas parciales: con series de Fourier y problemas de contorno. 3a ed. Madrid: Prentice Hall, 2003. ISBN 8420535346.

Complementary

- Garnir, H.G. Teoria de funciones: curso de análisis matemático de la Facultad de Ciencias de la Universidad de Lieja: tomo I. Barcelona: Técnicas Marcombo, 1966.
- Duvaut, G. Mécanique des milieux continus. Paris: Masson, 1990. ISBN 2225816581.
- Courant, R.; Hilbert, D. Methods of mathematical physics. New York [etc.]: Wiley, 1953-1962. ISBN 047017952X.
- Stakgold, I. Boundary value problems of mathematical physics. Philadelphia: SIAM, 2000. ISBN 0898714567.
- Encinas, A.M.; Rodellar, J. Curso de ecuaciones diferenciales en derivadas parciales. Barcelona: UPC - Campus Virtual Atenea, 2008.
- Peral, I. Primer curso de ecuaciones en derivadas parciales. Argentina [etc.]: Addison-Wesley, 1995. ISBN 0201653575.
- Marcellán, F.; Casasús, L.; Zarzo, A. Ecuaciones diferenciales : problemas lineales y aplicaciones. 1ª Edición. Madrid, [etc.]: McGraw-Hill, 1990. ISBN 8476155115.
- Nagle, R.K.; Saff, E.B.; Snider, A.D. Ecuaciones diferenciales : y problemas con valores en la frontera. México: Pearson Educacion, 2005. ISBN 970260592X.