Trabajo Grupo 4

PERMUTACIÓN 3

21 de diciembre de 2010

FONAMENTS MATEMÀTICS

Grau d'Enginyeria de la Construcció (E.T.S.E.C.C.P.B.)

1.- Calcular una primitiva de las siguientes funciones:

$(i)\displaystyle\int \frac{x}{\cos^2(x)}dx$ $\quad (ii)\displaystyle\int \frac{2(x-1)}{x^2-2x+3}dx$ $\quad (iii)\displaystyle\int \sqrt{8-x^2}dx$

$(iv)\displaystyle\int \frac{e^{\frac{1}{x^2}}}{x^3}dx$ $\quad (v)\displaystyle\int \frac{dx}{x^2+7}$ $\quad (vi)\displaystyle\int \frac{e^{\ln(x)}}{x}dx$

$(vii)\displaystyle\int \operatorname{sen}^3(x)dx$ $\quad (viii)\displaystyle\int x\,\cosh(x)\,dx$ $\quad (ix)\displaystyle\int \operatorname{cotg}^2(x)dx$

SOLUCIÓN

$$(i)\int \frac{x}{\cos^2(x)}dx = \left[\begin{array}{ll} u = x & \longrightarrow du = dx \\ dv = \dfrac{dx}{\cos^2(x)} & \longrightarrow v = \tan(x) \end{array}\right] = x\,\tan(x) - \int \tan(x)dx$$

$$= x\,\tan(x) - \int \frac{\operatorname{sen}(x)}{\cos(x)}dx = x\,\tan(x) + \ln|\cos(x)| + C$$

$$(ii)\int \frac{2(x-1)}{x^2-2x+3}dx = \ln|(x^2-2x+3)| + C$$

$$(i)\int \sqrt{8-x^2}\,dx = \left[\begin{array}{ll} x = 2\sqrt{2}\operatorname{sen}(t) & dx = 2\sqrt{2}\cos(t) \\ t = \operatorname{arc\,sen}\left(\dfrac{x}{2\sqrt{2}}\right) & \end{array}\right] = \int \sqrt{8-8\operatorname{sen}^2(t)}2\sqrt{2}\cos(t)\,dt$$

$$= 8\int \cos^2(t)dt = 4\int [1+\cos(2t)]dt = 4t + \operatorname{sen}(2t) + C$$

$$= 4\operatorname{arc\,sen}\left(\frac{x}{2\sqrt{2}}\right) + 2\operatorname{sen}\left(2\operatorname{arc\,sen}\left(\frac{x}{2\sqrt{2}}\right)\right) + C =$$

$$(iv)\int \frac{e^{\frac{1}{x^2}}}{x^3}dx = \left[\ t = \tfrac{1}{x^2} \quad dt = -\tfrac{2}{x^3}\,dx\ \right] = \int \frac{e^t}{x^3}\,\frac{x^3}{-2}dt = -\int \frac{e^t}{2}dt = -\frac{1}{2}\,e^t = -\frac{1}{2}\,e^{\frac{1}{x^2}} + C$$

$$(v)\int \frac{dx}{x^2+7} = \frac{1}{7}\int \frac{dx}{\left(\frac{x}{\sqrt{7}}\right)^2+1} = \frac{\sqrt{7}}{7}\operatorname{arc\,tg}\left(\frac{x}{\sqrt{7}}\right)+C$$

$$(vi)\int \frac{e^{\ln(x)}}{x}dx = \int \frac{x}{x}dx = x+C$$

$$(vii)\int \operatorname{sen}^3(x)dx = \int \operatorname{sen}(x)(1-\cos^2(x))dx = \int \operatorname{sen}(x)dx - \int \operatorname{sen}(x)\cos^2(x)dx = -\cos(x)+\frac{\cos^3(x)}{3}+C$$

$$(viii)\int x\,\cosh(x)\,dx = \left[\begin{array}{ll} u = x & \longrightarrow du = dx \\ dv = \cosh(x)\,dx & \longrightarrow v = \operatorname{senh}(x) \end{array}\right] = x\,\operatorname{senh}(x) - \int \operatorname{senh}(x)\,dx$$

$$= x\,\operatorname{senh}(x) - \cosh(x) + C$$

$$(ix)\int \operatorname{cotg}^2(x)dx = \int \frac{\cos^2(x)}{\operatorname{sen}^2(x)}dx = \int \frac{1-\operatorname{sen}^2(x)}{\operatorname{sen}^2(x)}dx = \int \frac{1}{\operatorname{sen}^2(x)}dx - \int dx = -\operatorname{cotg}(x) - x + C$$