Trabajo Grupo **4**

PERMUTACIÓN 2

20 de diciembre de 2010

FONAMENTS MATEMÀTICS

Grau d'Enginyeria de la Construcció (E.T.S.E.C.C.P.B.)

1.- Calcular una primitiva de las siguientes funciones:

$(i)\ \displaystyle\int e^x \cos x\, dx$ $\qquad$ $(ii)\ \displaystyle\int \frac{dx}{x\sqrt{1-\ln^2(x)}}$ $\qquad$ $(iii)\ \displaystyle\int 2\cosh^3 x\ \operatorname{senh} x\, dx$

$(iv)\ \displaystyle\int \frac{x^2}{1+x^6}\, dx$ $\qquad$ $(v)\ \displaystyle\int \sqrt{4-x^2}\ dx$ $\qquad$ $(vi)\ \displaystyle\int \frac{\operatorname{senh}\sqrt{x}}{\sqrt{x}}\ dx$

$(vii)\ \displaystyle\int \cos^3 x dx$ $\qquad$ $(viii)\ \displaystyle\int \frac{x}{e^{x^2}} dx$ $\qquad$ $(ix)\ \displaystyle\int \frac{\ln(x)}{x^4} dx$

SOLUCIÓN:

$$(i)\int e^x \cos x\, dx = \left[\begin{array}{ll} u=\cos x & \longrightarrow du=-\operatorname{sen} x\, dx \\ dv=e^x dx & \longrightarrow v=e^x \end{array}\right] = e^x\cos x + \int e^x\ \operatorname{sen} x\, dx$$

$$= \left[\begin{array}{ll} u=\operatorname{sen} x & \longrightarrow du=\cos x\, dx \\ dv=e^x dx & \longrightarrow v=e^x \end{array}\right] = e^x\cos x + e^x\ \operatorname{sen} x - \int e^x \cos x\, dx$$

Despejando obtenemos:

$$\int e^x \cos x\, dx = \frac{e^x(\cos x + \operatorname{sen} x)}{2} + C$$

$$(ii)\int \frac{dx}{x\sqrt{1-\ln^2(x)}} = \left[\ t=\ln(x) \quad \longrightarrow dt=\frac{dx}{x}\ \right] = \int \frac{dt}{\sqrt{1-t^2}} = \operatorname{arc\,sen}(t)+C = \operatorname{arc\,sen}(\ln(x))+C$$

$$(iii)\int 2\cosh^3 x\ \operatorname{senh} x\, dx = \frac{\cosh^4 x}{2} + C$$

$$(iv)\int \frac{x^2}{1+x^6}\, dx = \frac{1}{3}\int \frac{3x^2}{1+(x^3)^2}\, dx = \frac{\arctan(x^3)}{3} + C$$

$$(v)\int \sqrt{4-x^2}\ dx = \left[\begin{array}{ll} x=2\operatorname{sen} t & \longrightarrow dx=2\cos(t)dt \\ t=\operatorname{arc\,sen}\left(\frac{x}{2}\right) & \end{array}\right] = \int \sqrt{4-4\operatorname{sen}^2(t)}\, 2\cos(t)\ dt$$

$$= 2\int 2\sqrt{1-\operatorname{sen}^2(t)}\cos(t)\, dt = 4\int \cos^2(t)\, dt = 4\int \frac{1+\cos(2t)}{2}\, dt$$

$$= 2t + \operatorname{sen}(2t) + C = 2\operatorname{arc\,sen}\left(\frac{x}{2}\right) + \operatorname{sen}\left[2\operatorname{arc\,sen}\left(\frac{x}{2}\right)\right] + C$$

$$(vi)\int \frac{\operatorname{senh}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\int \frac{\operatorname{senh}\left(\sqrt{x}\right)}{2\sqrt{x}}\,dx = 2\cosh\left(\sqrt{x}\right)+C$$

$$(iv)\int \cos^3 x dx = \int \cos(x)\left(1-\operatorname{sen}^2(x)\right)dx = \int \cos(x)dx - \int \cos(x)\,\operatorname{sen}^2(x)dx = \operatorname{sen}(x) - \frac{\operatorname{sen}^3(x)}{3}+C$$

$$(viii)\int \frac{x}{e^{x^2}}dx = -\frac{1}{2}\int -2xe^{-x^2}dx = -\frac{1}{2}\,e^{-x^2}+C$$

$$(ix)\int \frac{\ln(x)}{x^4}dx = \left[\begin{array}{ll} u=\ln(x) & du=\dfrac{1}{x}dx \\ dv=\dfrac{1}{x^4}dx & v=-\dfrac{1}{3x^3} \end{array}\right] = -\frac{\ln(x)}{3x^3}+\frac{1}{3}\int \frac{1}{x^4}dx = -\frac{\ln(x)}{3x^3}-\frac{1}{9x^3}+C$$