Trabajo Grupo **4**

PERMUTACIÓN 1

16 de diciembre de 2010

FONAMENTS MATEMÀTICS

Grau d'Enginyeria de la Construcció (E.T.S.E.C.C.P.B.)

1.- Calcular una primitiva de las siguientes funciones:

$(i)\int \frac{x}{e^x}dx$ $(ii)\int \frac{x^2}{x^3-1}dx$ $(iii)\int \sin^2(x)dx$

$(iv)\int \frac{x}{\sqrt{x^2+1}}dx$ $(v)\int \frac{\ln(x)}{x^3}dx$ $(vi)\int \frac{e^{4x}+3}{e^{3x}}dx$

$(vii)\int \text{sen}^3(x)dx$ $(viii)\int x^2\,\text{sen}\,x dx$ $(ix)\int \frac{dx}{x^2+4}$

SOLUCIÓN:

$$i)\int \frac{x}{e^x}dx = \left[\begin{array}{ll} u=x & du=dx \\ dv=e^{-x}dx & v=-e^{-x}\end{array}\right] = -xe^{-x}+\int e^{-x}dx = -xe^{-x}-e^{-x}+C = -e^{-x}(x+1)+C$$

$$(ii)\int \frac{x^2}{x^3-1}dx = \left[\ t=x^3 \quad dt=3x^2dx\ \right] = \int \frac{1}{3}\frac{1}{t-1}dt = \frac{1}{3}\ln|(x^3-1)|+C$$

$$(iii)\int \text{sen}^2(x)dx = \int \frac{1-\cos(2x)}{2}dx = \frac{1}{2}\left(x-\frac{\text{sen}(2x)}{2}\right)+C$$

$$(iv)\int \frac{x}{\sqrt{x^2+1}}dx = \int \frac{2x}{2\sqrt{x^2+1}}dx = \sqrt{x^2+1}+C$$

$$(v)\int \frac{\ln(x)}{x^3}dx = \left[\begin{array}{ll} u=\ln(x) & du=\frac{1}{x}dx \\ dv=\frac{1}{x^3}dx & v=-\frac{1}{2x^2}\end{array}\right] = -\frac{\ln(x)}{2x^2}+\frac{1}{2}\int \frac{1}{x^3}dx = -\frac{\ln(x)}{2x^2}-\frac{1}{4x^2}+C$$

$$(vi)\int \frac{e^{4x}+3}{e^{3x}}dx = \left[\begin{array}{ll} t=e^x & dt=e^xdx \\ \frac{dt}{t}=dx & \end{array}\right] = \int \frac{t^4+3}{t^3}\frac{dt}{t} = \int \frac{t^4+3}{t^4}dt = \int dt+3\int \frac{1}{t^4}dt$$

$$= t-\frac{1}{t^3}+C = e^x-\frac{1}{e^{3x}}+C$$

Alternativamente,

$$\int \frac{e^{4x}+3}{e^{3x}}dx = \int \left(e^{x}+3e^{-3x}\right)dx = e^{x} - \frac{1}{e^{3x}} + C$$

$$(vii)\int \text{sen}^3(x)dx = \int \text{sen}(x)(1-\cos^2(x))dx = \int \text{sen}(x)dx - \int \text{sen}(x)\cos^2(x)dx = -\cos(x) + \frac{\cos^3(x)}{3} + C$$

$$(viii)\int x^2 \,\text{sen}\, x\; dx = \left[\begin{array}{ll} u = x^2 & \longrightarrow du = 2x\; dx \\ dv = \text{sen}\, x\; dx & \longrightarrow v = -\cos x \end{array}\right] = -x^2\cos x + \int 2x\cos x\; dx$$

$$= \left[\begin{array}{ll} u = 2x & \longrightarrow du = 2dx \\ dv = \cos x dx & \longrightarrow v = \text{sen}\, x \end{array}\right] =$$

$$= -x^2\cos x + 2x\,\text{sen}\, x - \int 2\,\text{sen}\, x dx = -x^2\cos x + 2x\,\text{sen}\, x + 2\cos x + C$$

$$(ix)\int \frac{dx}{x^2+4} = \frac{1}{4}\int \frac{dx}{\left(\frac{x}{2}\right)^2+1} = \frac{1}{2}\int \frac{1}{2}\frac{dx}{\left(\frac{x}{2}\right)^2+1} = \frac{1}{2}\,\text{arc tg}\left(\frac{x}{2}\right) + C$$